1 Theory of dual-field interferometry

Interferometry surpasses the fundamental resolution limits of imaging astronomy with single telescopes by probing higher spatial frequencies (see other lectures in these series, e.g. Haniff, 2007). While current ground-based telescope diameters, defining the angular resolution of the telescope, are limited to 8-10 m, depending on the exact design of the telescope, the interferometric angular resolution scales with the baseline, the distance between the co-phased telescopes. The current technical baseline length limitation for ground-based interferometric arrays is 1-2 orders of magnitude larger than the single telescopes diameter limit, explaining the superior angular resolution of an interferometer. Going to very long beam trains and delay lines does not only bear technical difficulties, but is also challenging from the astronomical point of view: An interferometer is most sensitive to sources which are of the size of the nominal resolution of the interferometer ($\lambda_{\rm obs}\, B_{\rm proj}^{-1}$, [2]) or smaller. That means the larger the baseline is, the smaller the source needs to be to produce a strong instrument response (remember the similarity theorem of the Fourier transform). But smaller sources are usually farther away and thus apparently fainter. In other words, sources which are so small that they would require kilometer-long baselines to be resolved are usually so faint that very large individual apertures are needed to deliver enough photons [1].

Dual-field interferometry and phase-referencing was invented to cope with the sensitivity limiting effects of the turbulent atmosphere, which so far has limited the application of the resolution advantage of long baseline interferometers in optical- and near-infrared astronomy. Thus dual-field interferometry does not reach beyond the fundamental resolution limits of interferometry but it helps reaching these limits on fainter objects in the reality of a ground-based telescope array. To understand this we focus on the visibility phase for a moment.

1.1 Phases in interferometry

An interferometric measurement aims at measuring the amplitude and phase of the target at a certain spatial frequency [2]. Measuring the amplitude or

[1] This is a difference to radio interferometry. A lot of very bright radio sources are very powerful non-thermal synchrotron sources in quasars which have source sizes by orders of magnitude smaller than any reasonable thermal sources of similar flux. The infrared sky however is dominated by thermal sources: stars and the surrounding, heated matter.

[2] The spatial frequency probed is given by the ratio $B_{\rm proj}\, \lambda_{\rm obs}^{-1}$ and has the dimension of inverse radians. $B_{\rm proj}$ is the projected baseline length at the moment of

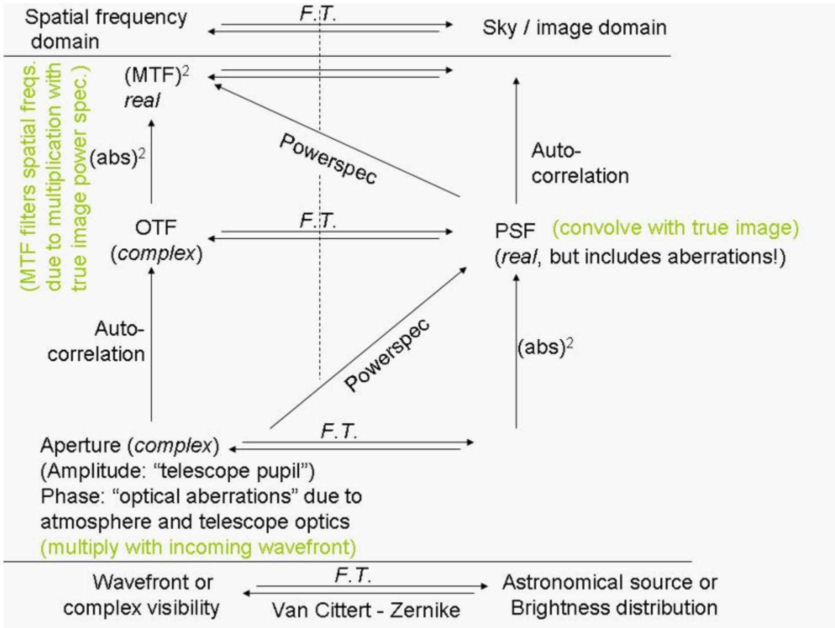

Fig. 1. Sketch describing how the Fourier transform connects the individual elements of an astronomical imaging process: aperture, optical transfer function (OTF), point spread function (PSF), modulation transfer function (MTF). It becomes obvious that the telescope aperture is a filter for certain spatial frequencies. Here *telescope* can similarly mean the circular aperture of a single telescope or the spotted aperture of a co-phased array.

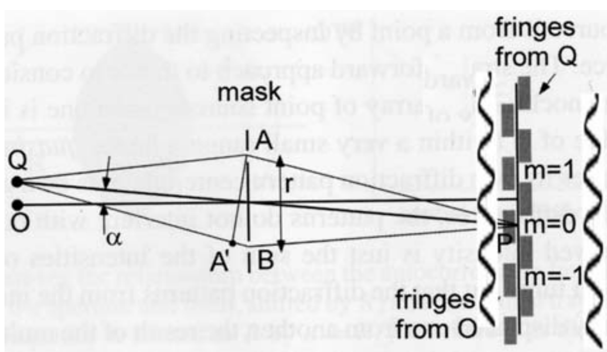

Fig. 2. Two-element interferometer seen as a double-slit experiment. Here the connection between *phase* of the fringe pattern and fringe location becomes visible. The target is simplified to two point sources Q and O, separated by the angle α. The optical path difference (OPD) equals AP-BP. And the differential OPD is geometrically connected to the angle α and to the difference in fringe positions, each given by the location of the intensity maximum, denoted with m=0 for the fringe from Q. This is the basis of interferometric astrometry. The slit distance r stands for the baseline. The angular distance α can be derived from the differential fringe position and the baseline length (the sketch is derived from Labeyrie et al., 2006).

power at this frequency is typically easier than measuring the phase. While the amplitude is a measure for the compactness of the source, the phase of the interferometric observable, the visibility, contains crucial information about the brightness *distribution* of the target, that is the details of the source structure. The phase of the spatial frequency is given by the shape of the target and is independent of time until the shape changes due to an astrophysical change in the source, e.g. the creation of a new dust shell. This connection between interferometric phase and imaging information has been introduced by other authors in the series (e.g. Monnier, 2007) and is most comprehensively described by the van Cittert-Zernike theorem. It states that the brightness distribution of a target can be analyzed by a two-dimensional Fourier-transformation, whereas a simple one-baseline interferometer, a co-phased two-telescope array, is *the* tool to measure one of the Fourier components at a time. This is nothing particularly surprising, since the Fourier transform plays a crucial role in *every* type of astronomical imaging, as summarized in Fig. 1.

The connection of visibility phase and imaging information can be understood immediately by acknowledging the role of the Fourier transform. Translating the position in the image space (that is "moving the star on the sky") is not changing the amplitude in visibility space (because the target still "looks the same"), but a linear phase slope is added to the complex visibility function. The meaning of the visibility phase as a way to locate the source in the sky with respect to a deliberately chosen phase center (typically a nearby bright star) is shown in Fig. 2 by the analogy between a two-element interferometer and imaging through a double-slit.

The interferometric phase or visibility phase basically encodes the optical path differences between two incoming rays of light. This leads immediately to the understanding of how the turbulent atmosphere hampers interferometric phase measurements: Different optical path lengths or different indices of refraction along different ways through the atmosphere are created by continuous temperature, density and compositional variations of the atmosphere. While temperature and density changes (e.g. due to winds) dominate the variation of the atmospheric index of refraction in the visible, the humidity becomes more and more important toward the longer wavelengths in the infrared. In particular the *changes* of humidity can be rather large in a short time due to the non-homogeneous distribution of water in the atmosphere (think of clouds).

The visibility phase of a target is only defined with respect to a phase center. This can be given by the fringe position [3] of another star, or by the fringe

observation, and λ_{obs} denotes the effective observing wavelength. The reciprocal of the spatial frequency, $\lambda_{\mathrm{obs}}\, B_{\mathrm{proj}}^{-1}$ is the nominal resolution of the interferometer.

[3] Note that *fringe position* usually refers to the OPD of the starlight, which has to be equalized by the delay lines to enable interference (Fig. 2). In the case of the KI, the delay lines consist of movable mirrors of which the actual position is known and

position of the same object but observed at a different baseline. The difficulty is to ensure that the measured phase, as a differential information, only traces the true geometrical delay, and is not altered by the turbulent atmosphere or instrument-induced phase changes.

Here we need to discuss two properties of the turbulent atmosphere which dominate the difficulty of a phase measurement, and therefore an interferometric measurement at all: piston stability (Sect. 1.2) and an-isopistonism (Sect. 1.3.

1.2 Piston stability

Fig. 3. The quasi-static long delay lines of the Keck Interferometer can correct for $\pm\,70\,\mathrm{m}$ of geometric delay.

The atmosphere adds a random phase to each beam due to continuously varying index of refraction. This phase, averaged over the telescope aperture, is called *piston*. Being constant over the aperture, the piston does not affect single telescope imaging (Fig. 1), but a differential piston between two telescopes changes the effective OPD and biases a phase (= astrometric) measurement.

continuously measured. Fig. 3 shows the long delay lines of the KI, which are a part of the delay line system. It is convenient to think of the fringe position as the physical position of the delay line reflectors, at which the center of the interferometric fringe pattern is found (see Fig. 4 in Haniff, 2007). However similarly to the phase, this is only a differential measure, and the zeropoint is arbitrary.

Although piston is only seen in interferometric data, it goes back to the same atmospheric turbulence responsible for the seeing. Therefore the magnitude and velocity of the differential piston variation is roughly correlated with the magnitude of seeing and the atmospheric coherence time. Since the fringe position is continuously varied by the differential piston, the integration time needs to be shorter than the timescale of the piston variation to avoid a blurring of the fringe pattern which will decrease the measured fringe contrast (Eq. 1). Too large a phase variation during an interferometric measurement, which takes a finite amount of time and therefore averages the visibility amplitude, reduces the fringe SNR in the following way

$$V_{\rm avg}^2 = V_0 \, \exp\left(-\sigma_{\rm rad}^2\right) \tag{1}$$

where V_0 and $V_{\rm avg}$ are the original and time-averaged visibility amplitudes, and $\sigma_{\rm rad}^2$ is the phase variance over the detector integration time.

That means, similar to speckle imaging, the signal-to-noise ratio (SNR) of the observable, the *correlated* flux, *decreases* with increasing integration time when the integration time is longer than timescale of piston stability (tens to hundreds of milliseconds in the infrared, depending on the exact observing wavelength and current atmospheric conditions). But the only way to estimate the current optimum position of the delay lines is to analyze the actual fringe. If the SNR of that fringe is not high enough to do so, the derived delay line position is faulty and the fringe is lost. Thus the changing atmospheric piston is limiting the integration time and the sensitivity of the interferometer. This is one of the two fundamental reasons why until today co-phased arrays are relatively insensitive and bound to study bright objects. The other reason is the low optical throughput of an interferometer, which originates in the need for 2-3 times as many reflecting / transmitting optics as needed for a single telescope imaging camera and in intensity losses due to the large distance between the primary mirror and the detector.

The only way to overcome this sensitivity problem is a dual-field interferometer which observes at the same time two nearby stars through similar atmospheric piston, and with one of them being bright enough to measure the fringe and preset the delay lines correctly. This fringe stabilization enabling longer coherent integration times and higher SNR on the second star is called *fringe-tracking based on phase-referencing*. It is the basis of the ASTRA project.

1.3 An-isopistonism

Similar to the effect of an-isoplanatism, which limits the correction of an adaptive optics (AO) experiment over the field-of-view, the differential piston correction of phase-referencing (PR) degrades with the separation distance between the two fields observed. If the two stars are too far away, the turbulence profiles along the individual lines of sight to each star are not correlated to each other anymore, and one cannot correct for the piston toward the fainter star B by measuring the piston toward the brighter star A. Star A is also called the *phase-reference*. The differential piston between the two stars should not increase to 1 rad or beyond to keep the SNR losses smaller than 20 % (Eq. 1). This limits the isopistonic angle even at the best sites (Mauna Kea, Cerro Paranal) to less than 30 arcsec at 2.2 μm (Esposito et al., 2000).

Knowing the synonymous meaning of phase and distance between the two (unresolved) stars, relates the atmospheric residual piston noise of a differential measurement to the astrometric precision. Shao & Colavita (1992) showed that in the narrow angle regime, where the differential piston noise is small, longer baselines B and smaller star-star separations θ decrease the impact of atmospheric piston on an interferometric astrometry measurement (astrometric error $\propto \theta B^{-2/3}$). Thus longer baselines are favorable for instruments like ASTRA. Also the outer scale of the atmospheric turbulence and its relation to the baseline relates to the astrometric noise. The longer the baseline is with respect to the outer scale, the smaller the astrometric error is as derived from the differential piston noise (see Fig. 3 & 4 in Shao & Colavita, 1992).

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